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Optimization of Multiple-Panel Compliant Walls for Delay of Laminar-Turbulent Transition

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Abstract

PREVIOUS theoretical work has led to the development of methods for optimizing compliant-wall properties to achieve the greatest possible transition delay for flat-plate boundary-layer flows. It was found that the optimum wall properties for locally reducing the growth of disturbances depend quite strongly on the Reynolds number. This suggests that it would be advantageous to use multiple-panel walls with each compliant panel optimized for a particular Reynolds number range. Accordingly, the optimization procedure is extended to two-panel compliant walls, and the optimum wall properties are determined that correspond to the greatest transition delay. It is found that, based on a conservative value of $n = 7$, the e^n method predicts that the greatest transitional Reynolds number achievable using single- and two-panel compliant walls is respectively 4.6 and 6.05 times the rigid-wall value. The corresponding wall properties are readily realizable for operation in water, but are probably not feasible in air.

Contents

There is little doubt that for boundary layers in water flow the use of appropriately designed compliant walls can lead to substantial postponement of laminar-turbulent transition. This has been established both theoretically and experimentally.[1] It is quite likely that doubts remain for some readers as to whether it is really possible to manufacture compliant walls with the properties required to achieve substantial transition delay at the flow speeds typical of marine applications of practical interest. It is hoped that the present paper will help to dispel some of these doubts. However, it is accepted that nothing short of actual application of a compliant wall at the appropriate speed and in the appropriate marine environment will be fully convincing. Meanwhile, the emphasis of the research in this area is now passing from the question of whether or not compliant walls are a viable method of achieving transition delay to the development of design methods with the object of determining the optimal wall properties for achieving the greatest possible transition delay. An optimization methodology has been developed[2] for a class of anisotropic and isotropic compliant walls. In the present paper this is extended to compliant walls consisting of two isotropic panels in series (see Fig. 1).

Some aspects of the optimization procedure are quite general. However, to obtain results, a particular compliant-wall model must be used. For the present work the plate-spring model developed by Carpenter and Garrad[3] is used. This model is chosen because it is reasonably representative of a wide range of compliant walls and because a number of analytical results are available to make the optimization process tractable. It is assumed that the wall is driven by two-dimensional disturbances and the perpendicular surface displacement η is governed by the following equation of motion:

$$\rho_m b \frac{\partial^2 \eta}{\partial t^2} + B \frac{\partial^4 \eta}{\partial x^4} + K\eta = p_s - p_e \tag{1}$$

where ρ_m is the density of the plate material, b is the plate thickness, B is the flexural rigidity of the plate, and K is the effective spring stiffness per unit width. The symbols p_e and p_s are, respectively, the perturbations in dynamic pressure acting on the plate from above and below. The flexural rigidity is given by

$$B = \frac{Eb^3}{12(1 - \nu_P^2)} \tag{2}$$

where E and ν_P are, respectively, the elastic modulus and Poisson ratio of the plate material. For the elastomeric materials typically used for compliant walls ν_P is close to 0.5. Viscoelastic material properties can be accounted for by introducing complex E and K (Ref. 3).

Regarding the development of disturbances in the boundary layer, conventional linear boundary-layer stability theory is followed. The formulation and numerical solution of the eigenvalue problem is identical to that described by Carpenter and Morris,[2] and only a brief outline is given in the following.

The wall displacement, disturbance velocity components and pressure are written in the usual two-dimensional traveling-wave form:

$$\{\eta, u, v, p\} = \{\hat{\eta}, \hat{u}, \hat{v}, \hat{p}\}\exp[i(\alpha x - \omega t)] \tag{3}$$

where α is the complex wave number, with $-\alpha_i$ representing the growth rate of the disturbance, and ω is the frequency. The usual quasiparallel approximation is made for the boundary-layer flow so that the disturbance velocity amplitude $\hat{v}$ is governed by the Orr-Sommerfeld equation. The undisturbed streamwise velocity profile is given by the Blasius velocity profile. The boundary conditions applied to $\hat{v}$ at the outer edge of the boundary layer are the same as those for a rigid surface. For a compliant wall in general it is necessary to solve the equation(s) of motion for the wall and the Orr-Sommerfeld equation simultaneously, coupling them with interfacial conditions. These conditions require that the velocity and stress be continuous between fluid and solid at the compliant surface.

It is well known that, in theory, Tollmein-Schlichting waves (TSW) can be completely suppressed by using a sufficiently compliant wall. What prevents this course of action from succeeding in practice is the appearance of other modes of instability. Two additional modes of instability are commonly

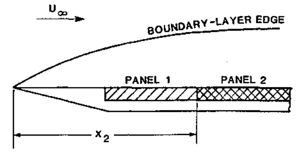

Fig. 1 Schematic of a two-panel compliant wall.

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Table 1 Optimal wall properties for water flow at 20 m/s

Panel	$\bar{\alpha}_d$	b, mm	E, MN/m^2	K_E, GN/m^3
1	39.3×10^{-6}	1.10	1.385	0.236
2	24.6×10^{-6}	1.76	1.385	0.148

Table 2 Values of C_{Df}

Case	Re_L	$(Re_x)_t$	C_{Df}
Rigid wall	10×10^6	2.25×10^6	0.00252
	20×10^6		0.00248
	50×10^6		0.00227
Optimal single-panel wall	10×10^6	10.36×10^6	0.00042
	20×10^6		0.00158
	50×10^6		0.00197
Optimal two-panel wall	10×10^6	13.62×10^6	0.00042
	20×10^6		0.00118
	50×10^6		0.00184

found in boundary layers over compliant walls. These are both essentially instabilities in the wall itself. One, termed traveling-wave flutter (TWF), is convective in nature. The other, termed divergence, is absolute. It is the appearance of these two instabilities that limits the transition delay attainable by use of the compliant walls. In fact, it is now known that the TWF instability, rather than the TSW, was the route to transition for many of the compliant panels studied experimentally. The two main wall instabilities are more fully described in Ref. 1.

The essence of the optimization scheme is to require the compliant wall to be marginally stable with respect to both TWF and divergence. For single-panel nondissipative walls this gives a one-parameter family of wall properties. It is mathematically convenient to choose the nondimensional critical wave number for divergence, denoted by $\bar{\alpha}_d$, as this single parameter.[2] To determine the wall properties corresponding to the greatest possible transition delay, the well-known e^n method of transition prediction is used in Ref. 2 for a range of values of $\bar{\alpha}_d$. The method is based on calculating the total amplification ratio at a given streamwise position, x (or corresponding value of Re, the Reynolds number based on boundary-layer displacement thickness) of the TSW with a specified frequency.

The aim is to find the value of $\bar{\alpha}_d$ for which the critical value of n is reached at the greatest possible Reynolds number. Although much progress has been made recently, little is known about the nonlinear regime of transition over compliant walls. For this reason the rather conservative value of $n = 7$ (which corresponds approximately to the limits of the linear regime of transition in a low-disturbance environment) is chosen instead of the usual value of $n \simeq 10$. The maximum amplification envelopes, obtained using the e^n method and presented in Ref. 2, are markedly sigmoid in form, indicating that the optimal wall properties for reducing TSW growth rate vary very considerably with Reynolds number. This strongly suggests that the use of a multiple-panel compliant wall, with each panel optimized for a particular range of Reynolds number, is likely to lead to even greater transition delays. However, the more panels there are, the greater the problems of optimizing the wall properties. Accordingly, the results in the present paper are confined to a two-panel compliant wall.

For nondissipative two-panel walls the disposable wall parameters increase to three: $\bar{\alpha}_{d1}$, $\bar{\alpha}_{d2}$, and Re_2 (the value of Re corresponding to x_2 in Fig. 1). However, it can be shown that the best single-panel compliant wall is the best choice for panel 1. This fixes the value of $\bar{\alpha}_{d1}$, leaving α_{d2} and Re_2 to be varied to find the best two-panel wall. It is found that the greatest transition delay is obtained with a combination of $\bar{\alpha}_{d2} = 0.625\bar{\alpha}_{d1}$ and Re_2 taking a value slightly in excess of 4000.

Single- and two-panel compliant walls with viscoelastic damping were also investigated. It was found that the level of damping should be as low as possible to achieve the best transition-delaying performance. Nevertheless, a very considerable transition-delaying performance can still be achieved when damping is present.

Once the value of $\bar{\alpha}_d$ is determined the other nondimensional wall parameters can be calculated. However, to determine the actual material properties of the wall it is necessary to specify the density and Poisson ratio of the wall material and the flow parameters. For illustrative purposes a water flow of 20 m/s is assumed. The material is assumed to be an elastomer (e.g., silicone or natural rubber), with a density and Poisson ratio of 1000 kg/m^3 and 0.5, respectively. The corresponding optimum wall properties for a two-panel compliant wall are given in Table 1. Note that the entries for panel 1 correspond to the optimal properties for a single-panel wall.

The wall properties given in Table 1 are perfectly feasible since they are quite close to the estimates made in Ref. 3 for the properties of the original Kramer[4] compliant coatings. It can be shown that $b \propto 1/U_\infty$, thus, there may be difficulties in realizing these optimal walls in practice at much higher flow speeds. In airflows, because of the much lower density and somewhat higher kinematic viscosity, the values given in Table 1 for b would need to be multiplied by a factor of 0.02. Thus, even at the low speed of 20 m/s very thin membranes are required, and the use of compliant walls to produce substantial transition delays in airflows is unlikely to be practicable.

What sort of drag reductions are feasible with the present optimum single- and two-panel compliant walls? The range of Re_L (Reynolds number based on surface length) for certain applications are approximately as follows: 5 to 20×10^6 for hydrofoils, 20 to 50×10^6 for small submarines, and 40 to 70×10^6 for torpedos. The results in Table 2 were obtained by use of standard methods for calculating the skin-friction drag coefficient C_{Df} for a mixed laminar-turbulent boundary layer on a flat plate. Values for the transitional Reynolds number, $(Re_x)_t$, are also given in Table 2. It can be seen that a considerable benefit is predicted in terms of drag when compliant walls are used for $Re_L \leq 20 \times 10^6$. It should also be remembered that the estimates used for predicting the transitional Reynolds number $(Re_x)_t$ are based on the conservative value of $n = 7$. This may be an unduly restrictive assumption in view of the recent results of direct numerical simulations by Metcalfe et al.[5] showing that the benefits of wall compliance extend well into the nonlinear regime of transition.

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